Quantum Circuit Analysis with Model Counting (#SAT)

Santiago $\longleftrightarrow$ Leiden 2026 • Lecture 2 of 3

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Santiago 2026

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The Netherlands

Santiago 2026 — a three-lecture series

1. Demystifying Quantum Computing — *quantum computing 101*
2. **Quantum Circuit Analysis with #SAT** — *model counting*
3. Knowledge Representation for Quantum Computing — *knowledge representation*

Classical SAT solvers already crack open problems in math

MATHEMATICS

Maths proof smashes size record

Supercomputer produces a 200-terabyte proof — but is it really mathematics?

BY EVELYN LAMB

Three computer scientists have announced the largest-ever mathematical proof: a file that comes in at a whopping 200 terabytes, equivalent to all the digitized text held by the US Library of Congress. The researchers have created[1] a 68-gigabyte compressed version of their solution — which would allow anyone with about 30,000 hours of spare processor time to download, reconstruct and verify it — but a human could never hope to read through it.

Computer-assisted proofs too large to be directly verifiable by humans have become common, as have computers that solve problems in combinatorics — the study of finite discrete structures — by checking through umpteen individual cases. Still, "200 terabytes is unbelievable", says Ronald Graham, a mathematician at the University of California, San Diego. The previous record-holder is thought to be a 13-gigabyte proof[2], published in 2014.

The puzzle that required the 200-terabyte proof, called the Boolean Pythagorean triples problem, has troubled mathematicians for decades. In the 1980s, Graham offered a prize of US$100 for anyone who could solve it. (He presented the cheque to one of the three computer scientists, Marijn Heule of the University of Texas at Austin, last month.) The problem asks whether it is possible to colour each positive integer either red or blue, so that no trio of integers a, b and c that satisfy Pythagoras' famous equation $a^2+b^2=c^2$ are all the same colour. For example, for the Pythagorean triple 3, 4 and 5, if 3 and 5 were blue, 4 would have to be red.

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Neanderthals built cave structures — and no one knows why

MORE NEWS

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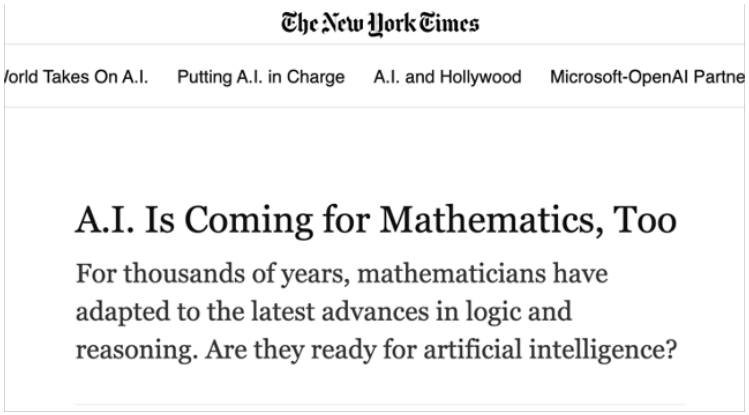
NATURE PODCAST

The genetics of moth evolution, Earth's core conundrum and Pluto's polygonal

Nature



The New York Times

World Takes On A.I. | Putting A.I. in Charge | A.I. and Hollywood | Microsoft-OpenAI Partne

A.I. Is Coming for Mathematics, Too

For thousands of years, mathematicians have adapted to the latest advances in logic and reasoning. Are they ready for artificial intelligence?

New York Times

[1] M. J. Heule and O. Kullmann, "The science of brute force," *Communications of the ACM*, vol. 60, 2017.

[2] M. J. Heule, O. Kullmann, and V. W. Marek, "Solving and verifying the boolean pythagorean triples problem via cube-and-conquer," in *International Conference on Theory and Applications of Satisfiability Testing*, Springer, 2016.

[3] M. Heule, "Schur number five," in *Proceedings of the AAAI Conference on Artificial Intelligence*, vol. 32, 2018.

The *same* classical SAT / #SAT solvers
now simulate, verify, and synthesize quantum circuits.

How can that possibly work?

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How can that possibly work?

Spoiler: *one* Boolean encoding — going quantum only changes the weights.

SAT-based Quantum Circuit Simulation

Encoding Quantum Circuit Semantics in SAT

- SAT encoding of Clifford circuits [1] ✓scalable / ✗non-universal

[1] L. Berent, L. Burgholzer, and R. Wille, "Towards a SAT Encoding for Quantum Circuits," in *SAT*, vol. 236, 2022.

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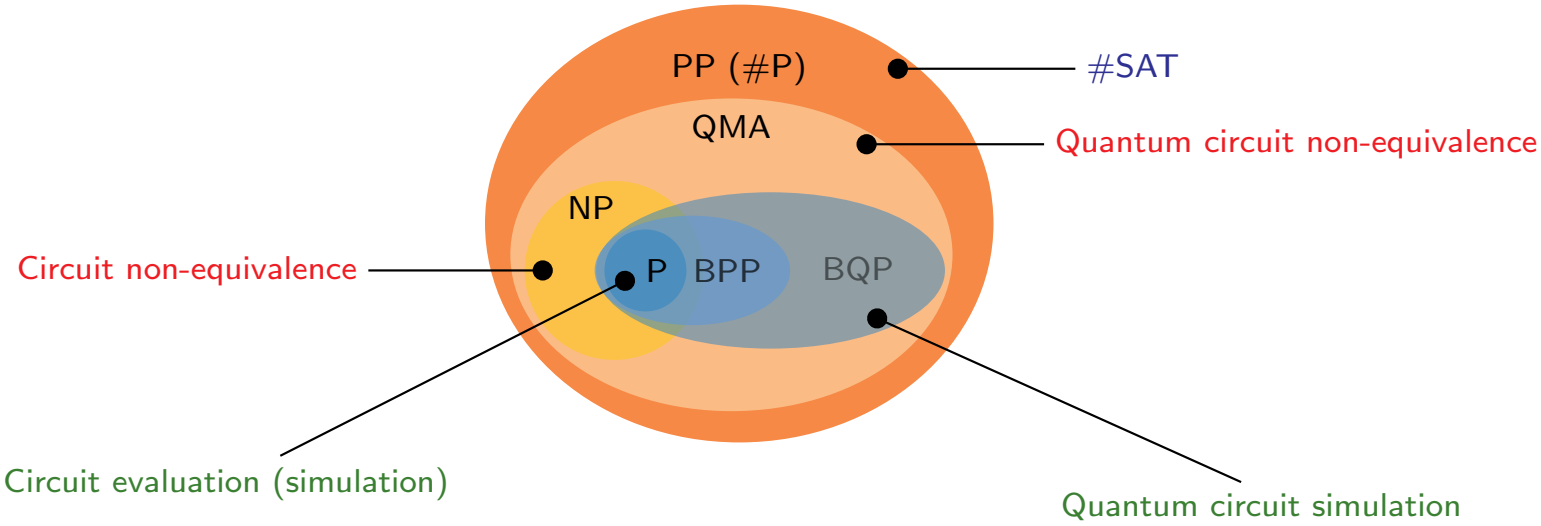

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Roadmap

- SAT and #SAT (model counting)
- Encoding quantum circuits in logic
- Application: Simulation, Equivalence checking, Verification, Synthesis
- Different encodings and performance

SAT = DPLL + CDCL

```
1:  procedure DPLL(φ in CNF)
2:      UnitPropagation(φ)
3:      if φ is empty CNF then
4:          return SAT
5:      if empty clause in φ then
6:          return UNSAT
7:      Select x ∈ vars(φ)
8:      Select v ∈ {0, 1}
9:      if DPLL(φ[x := v]) then
10:         return SAT
11:     else
12:         return DPLL(φ[x := 1 − v])
```

[DPLL = Davis, Putnam, Logemann & Loveland]

Implication Graph

$$\{\overbrace{19\bar{2}}^{c_1},\ \overbrace{1\bar{3}}^{c_2},\ \overbrace{234}^{c_3},\ \overbrace{\bar{4}\bar{5}}^{c_4},\ \overbrace{8\bar{4}\bar{6}}^{c_5},\ \overbrace{56}^{c_6},\ \cdots\}$$

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$$1 \qquad \boxed{\bar{8}}$$

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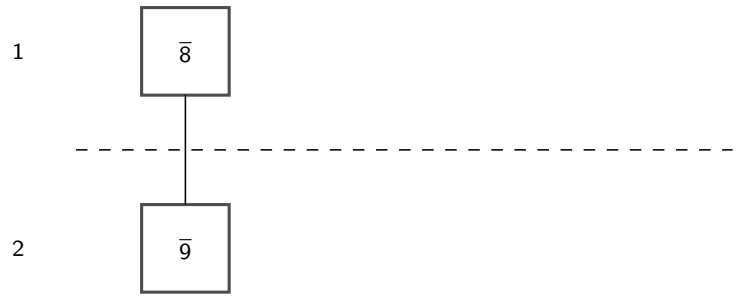

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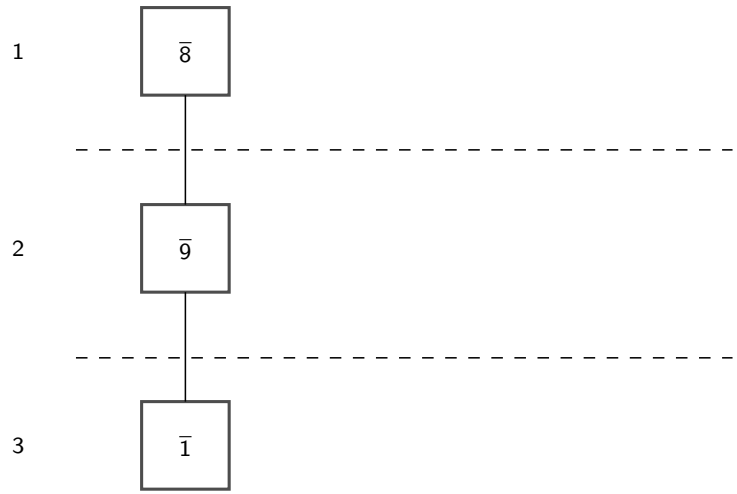

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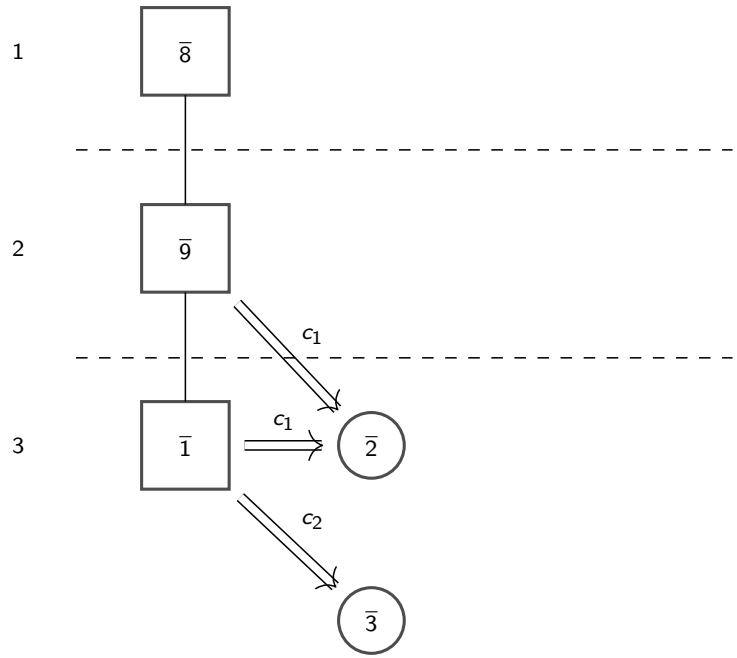

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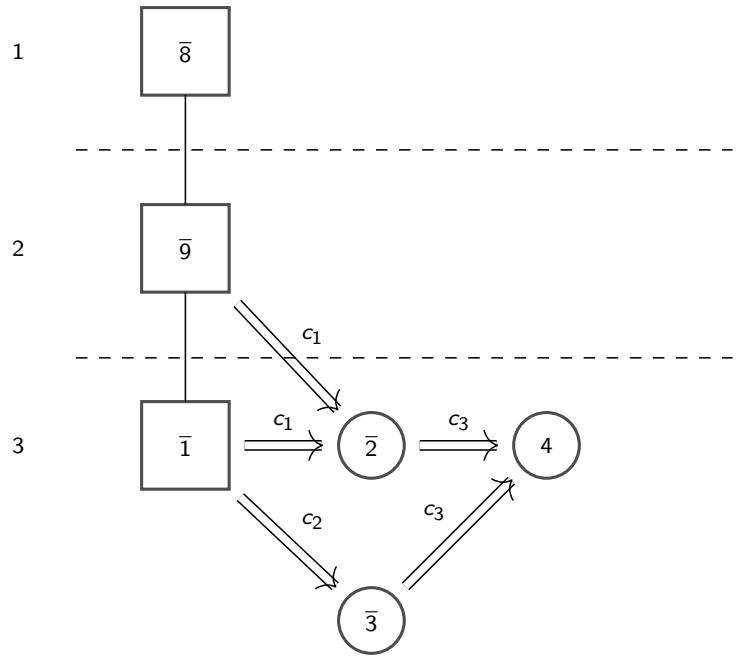

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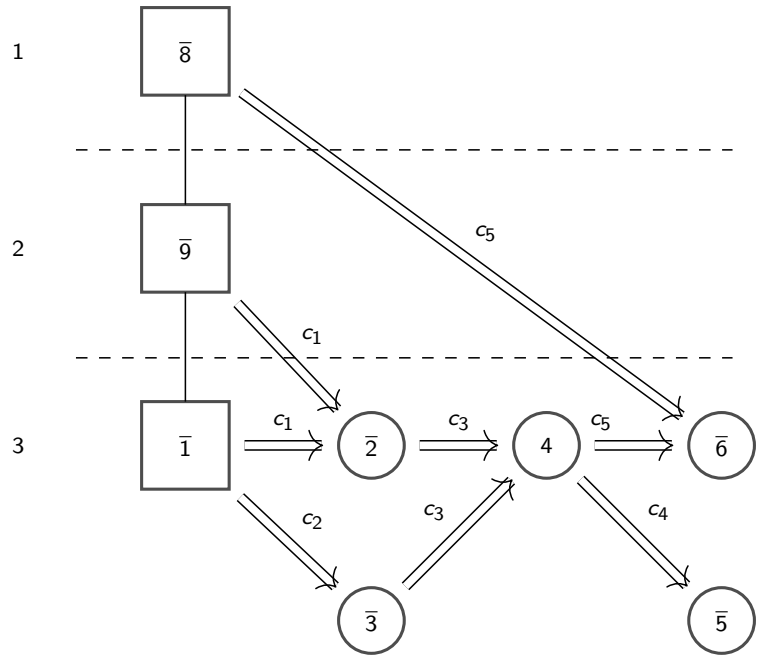

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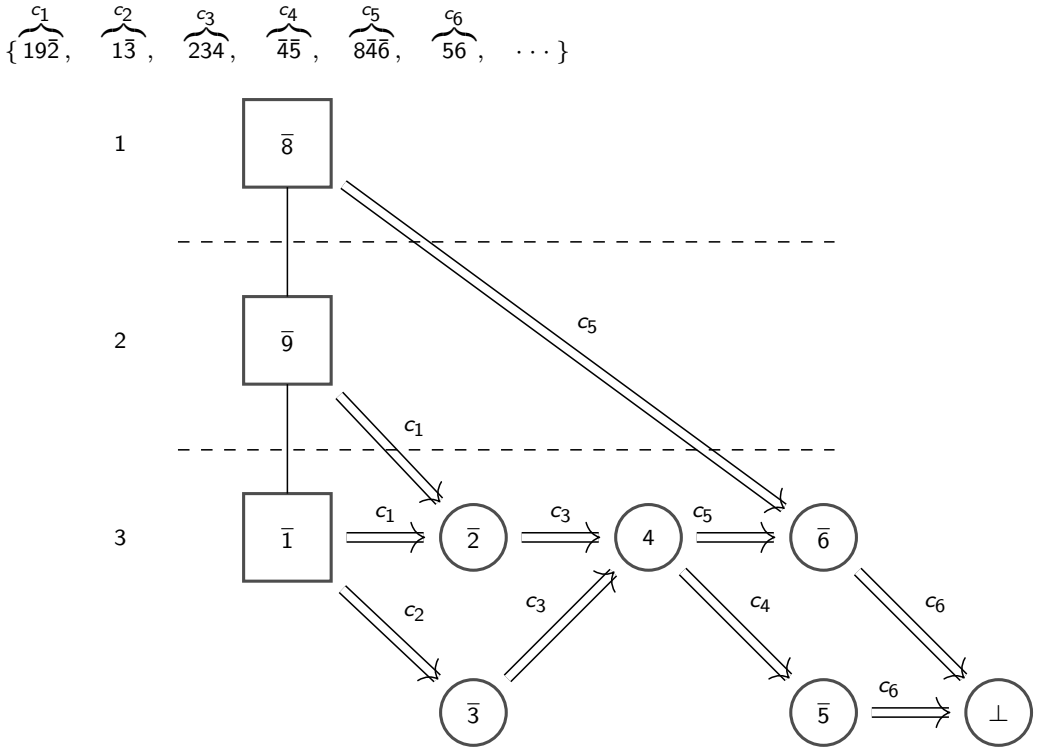
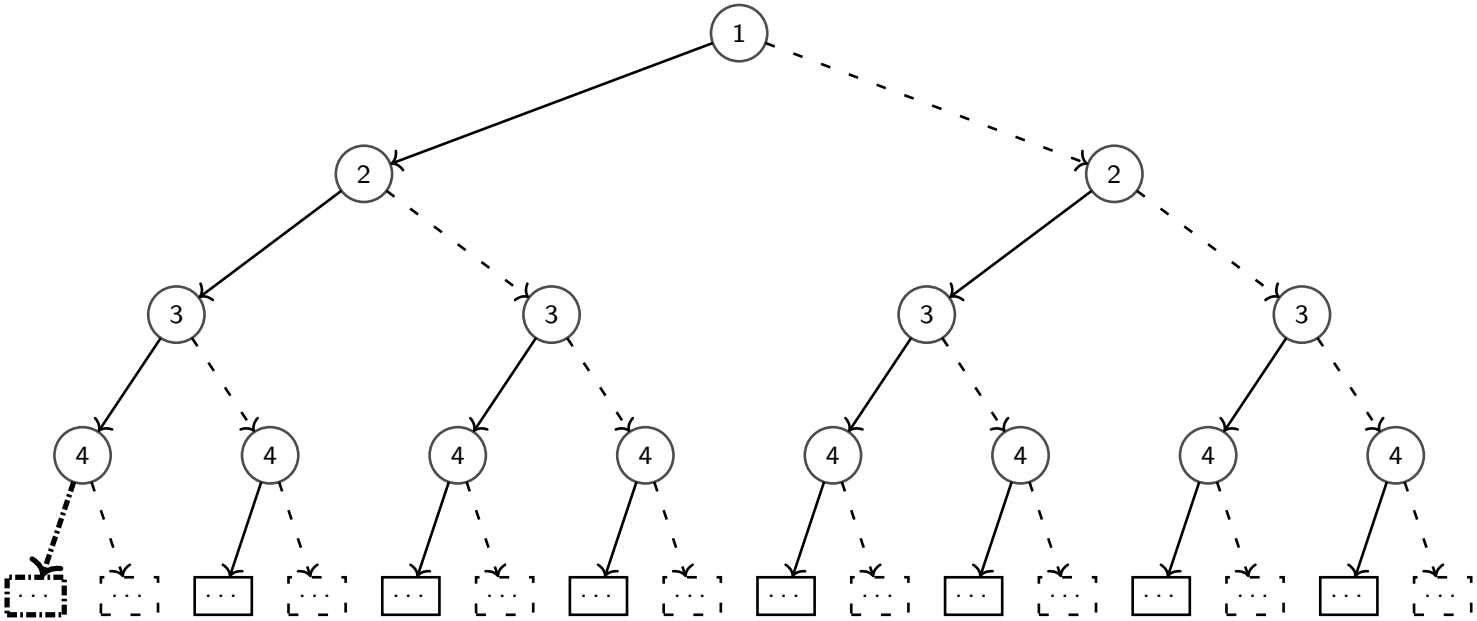


We may 'learn' a clause $C = 5 \vee 6$ or $C = \overline{4} \vee 6$

Constraint-Driven Clause Learning

$\varphi := \varphi \wedge c$ where c is a 'learned clause'



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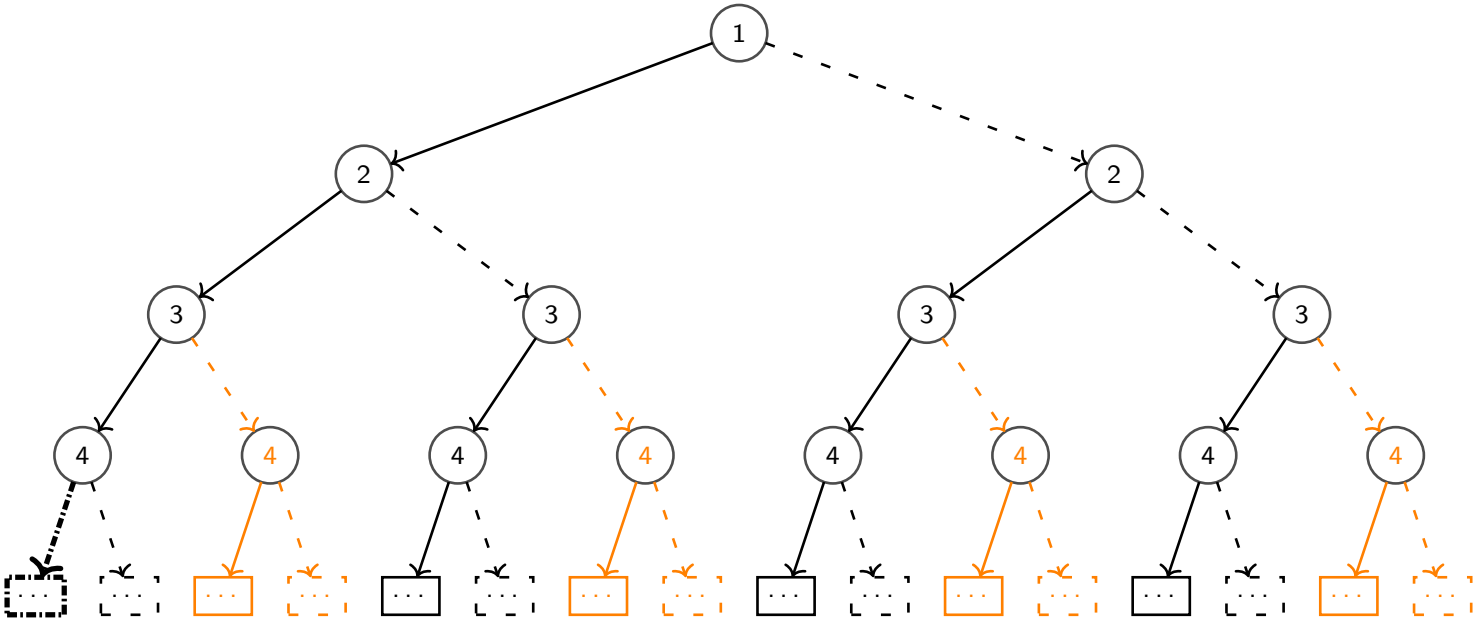

Proof theoretical results show exponential gains.

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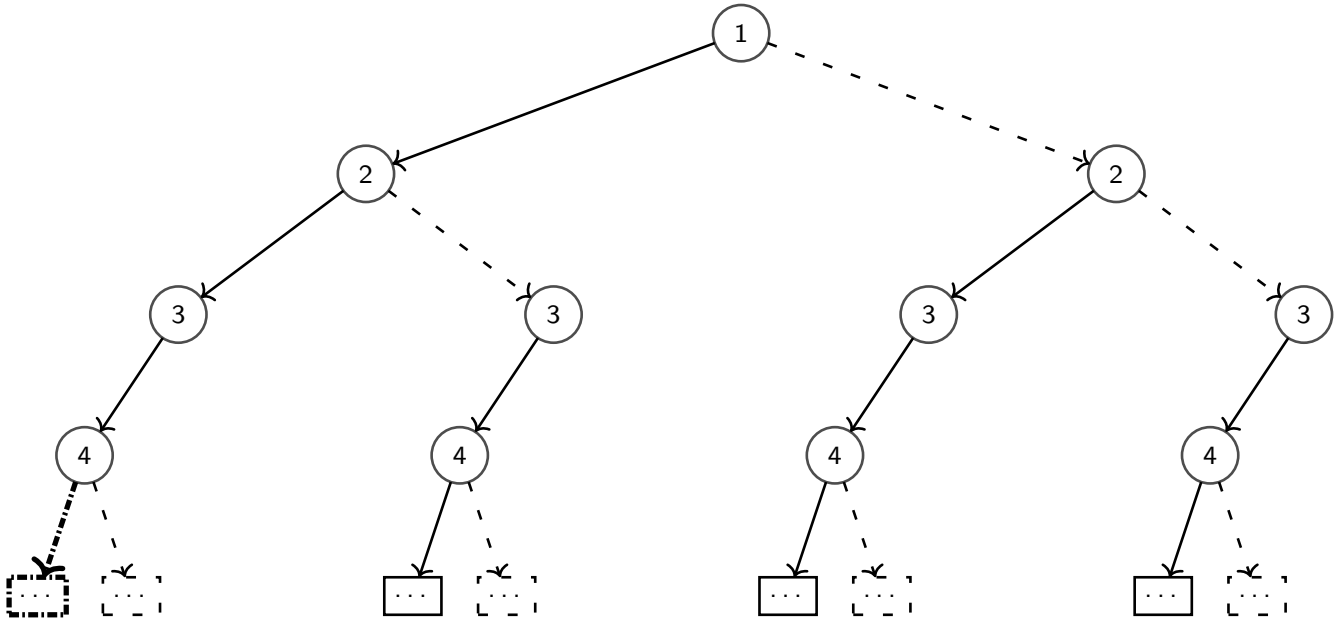

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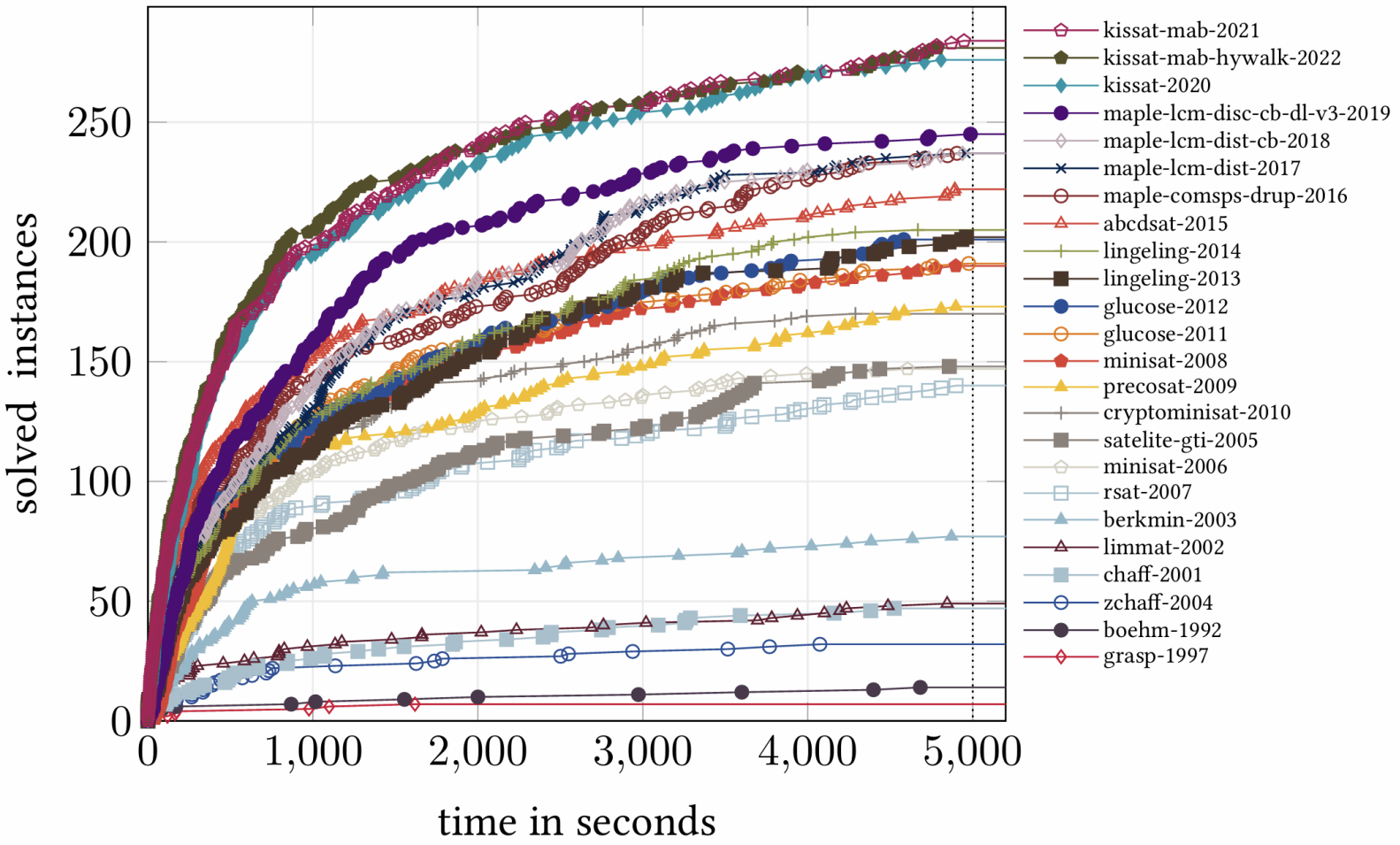
SAT Solvers Are Fast in Practice



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Example (Weighted Model Counting)

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Satisfying assignments:
$SAT(F) = \{ \{ a \leftarrow 0, b \leftarrow 1 \}, \{ a \leftarrow 1, b \leftarrow 0 \}, \{ a \leftarrow 1, b \leftarrow 1 \} \} = \{ \bar{a}b, a\bar{b}, ab \}$

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$\bar{a}b$	$2/3 \cdot 1$
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$$\text{ModelCount}_W(F) = \frac{4}{3} = \overbrace{\sum_{\vec{x} \in F^{-1}(1)}}^{\text{satisfying assignment}} \overbrace{\prod_{x_i \in \vec{x}} W(x_i)}^{\text{weight of assignment}}$$

Component caching

```
1:  procedure #DPLL(φ)
2:      φ ← UnitPropagation(φ)
3:      if φ = ∅ then
4:          return 1
5:      if ∅ ∈ φ then
6:          return 0
7:      if Cached(φ) then
8:          return Lookup(φ)
9:      if φ = φ1 ∧ φ2 with disjoint variables then
10:         return #DPLL(φ1) · #DPLL(φ2)
11:     x ← SelectVariable(φ)
12:     c ← #DPLL(φ[x := 0]) + #DPLL(φ[x := 1])
13:     Store(φ, c)
14:     return c
```

[1]

[1] M. Thurley, "Sharpsat–counting models with advanced component caching and implicit bcp," in *International Conference on Theory and Applications of Satisfiability Testing*, Springer, 2006.

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Encoding quantum states as Boolean formula

$|0\rangle$

$|1\rangle$

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- Step 1: Encode each term as an assignment to a set of variables

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$\frac{1}{\sqrt{2}}\|0\rangle + \frac{1}{\sqrt{2}}\|1\rangle$	$\left\{\{x \leftarrow 0, \text{“weight } \frac{1}{\sqrt{2}}\text{”}\}, \{x \leftarrow 1, \text{“weight } \frac{1}{\sqrt{2}}\text{”}\}\right\}$	

Encoding quantum states as Boolean formula

- Step 1: Encode each term as an assignment to a set of variables
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- Step 3: Incorporate weight by adding additional variable h with

$$W(z) = \begin{cases} \frac{1}{\sqrt{2}} & \text{if } z = \bar{h} \\ -\frac{1}{\sqrt{2}} & \text{if } z = h \\ 1 & \text{if } z \text{ represents any other literal} \end{cases}$$

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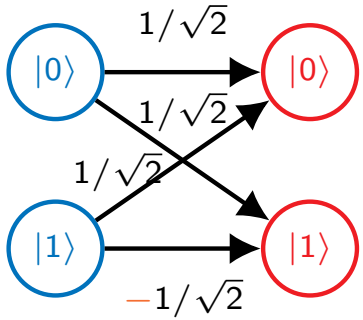
Encoding quantum gates

Goal: find formula $F_H(x, x', \quad)$ for the gate $H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

Encoding quantum gates

Goal: find formula $F_H(x, x', \underbrace{h_1, h_2, \ldots}_{\text{auxiliary variables}})$ for the gate $H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

$H(|x\rangle, |x'\rangle) =$



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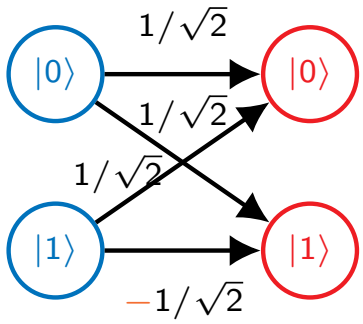
Goal: find formula $F_H(x, x', \underbrace{h_1, h_2, \ldots}_{\text{auxiliary variables}})$ for the gate $H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

$H(|x\rangle, |x'\rangle) =$



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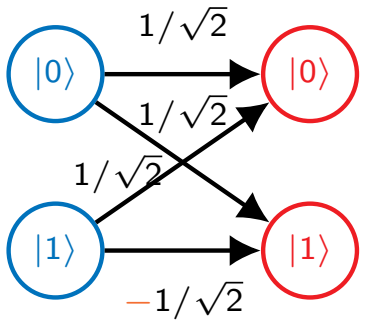

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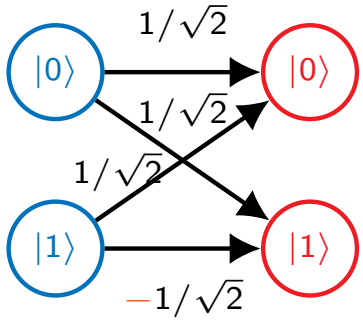

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Similarly, we can find formulae for other gates (including fixed rotations).

(For $H, CNOT$ and T is sufficient as these are universal; Solovey-Kitaev Thm)

Sequential and parallel composition of gates

Using F_H, F_{CNOT}, F_T for encoding the output state of a circuit C:

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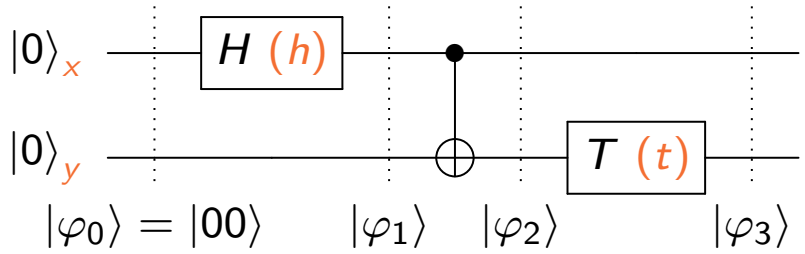

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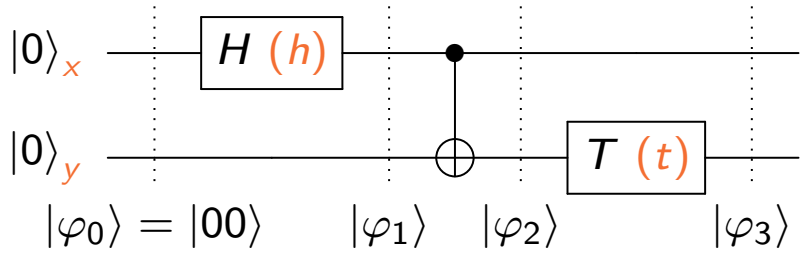

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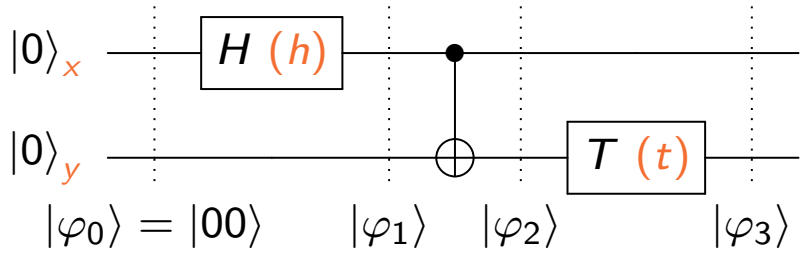

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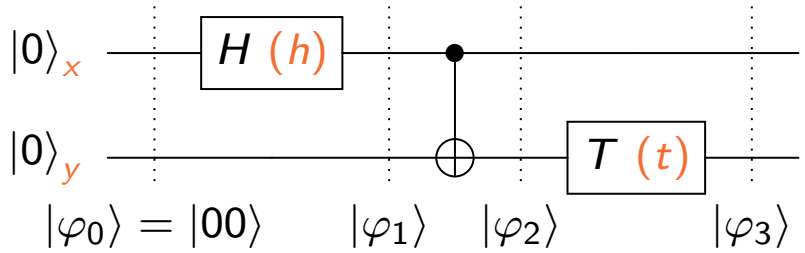

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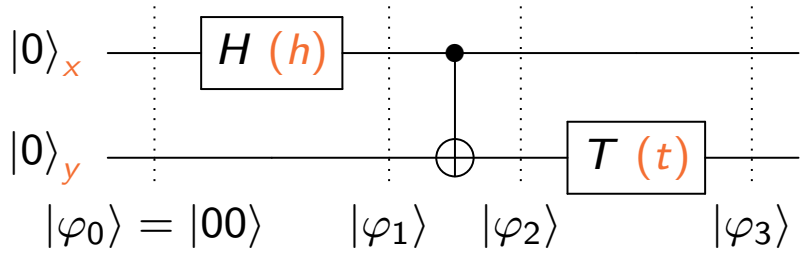

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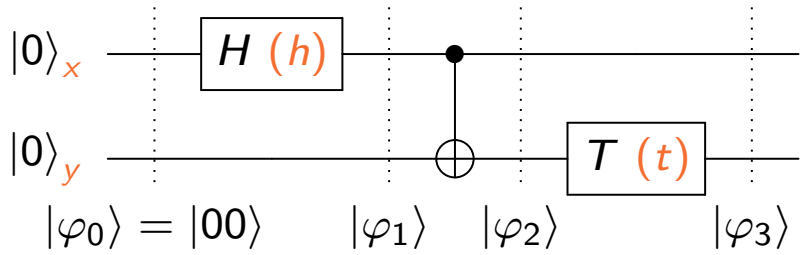

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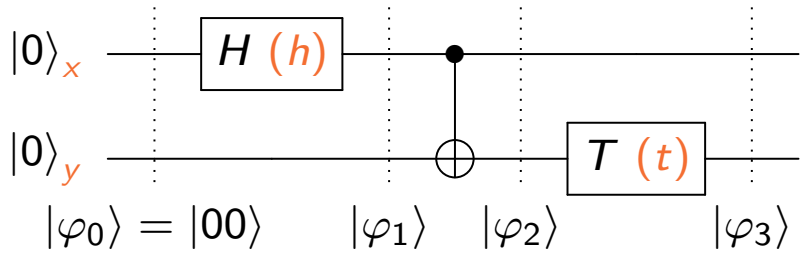

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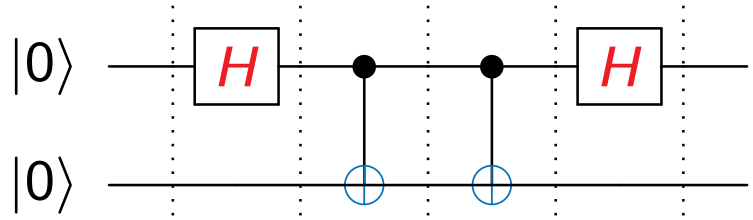
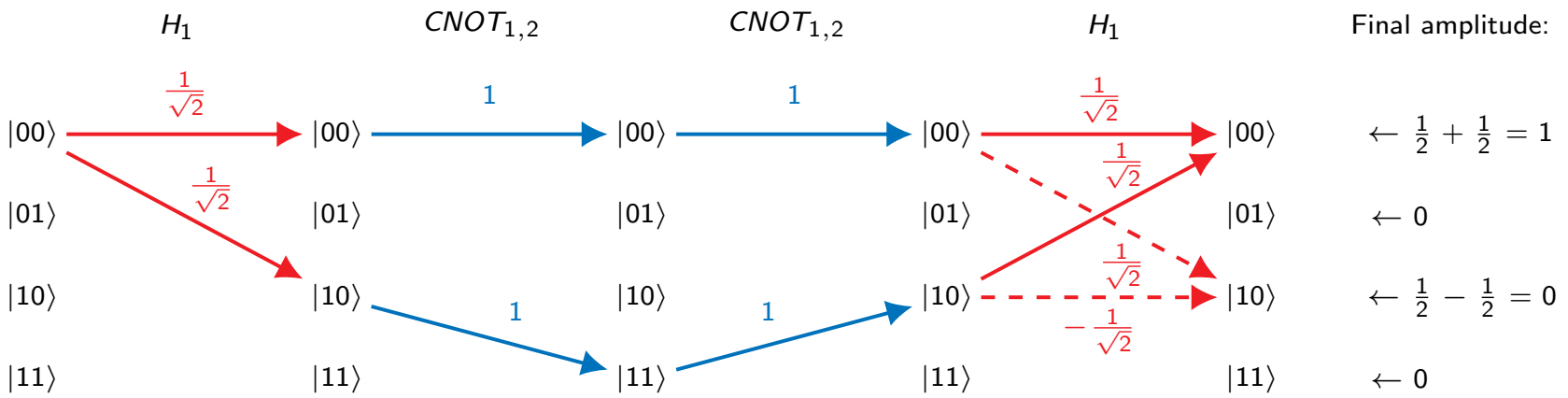
$$\#SAT_W(F_C \quad \wedge \quad x_3 = a \ \wedge \ y_3 = b) \quad = \quad \langle ab|\varphi_3\rangle$$

The Model Counter Computer Path Sums



$|\varphi_0\rangle$ $|\varphi_1\rangle$ $|\varphi_2\rangle$ $|\varphi_3\rangle$ $|\varphi_4\rangle$



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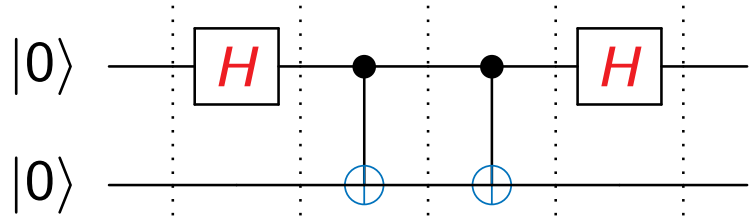

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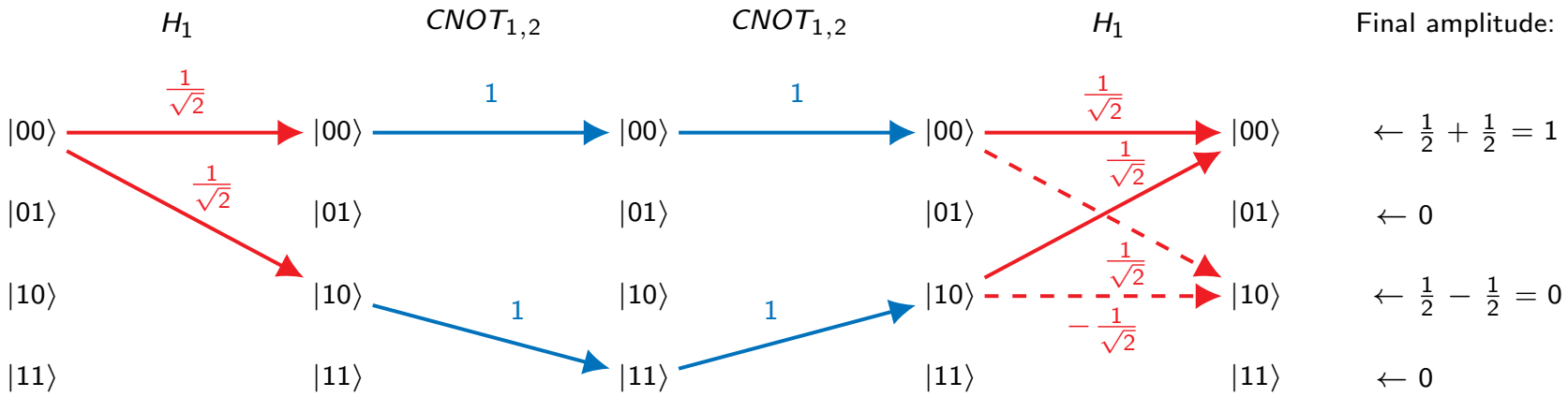

In WMC, the circuit above has four satisfying assignments:

$$\{h'\overline{h}x_1\overline{y}_1x_2y_2x_3\overline{y}_3x_4\overline{y}_4 \equiv -\tfrac{1}{2}\,|10\rangle\,, \quad \overline{h}'\overline{h}x_1\overline{y}_1x_2y_2x_3\overline{y}_3\overline{x}_4\overline{y}_4 \equiv \tfrac{1}{2}\,|00\rangle\,,$$
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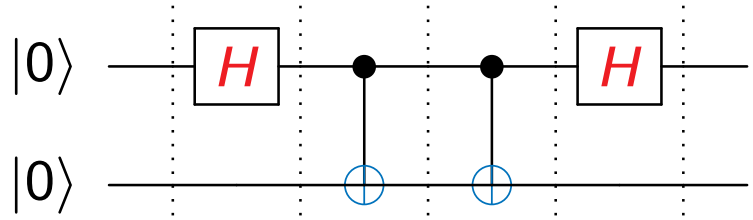

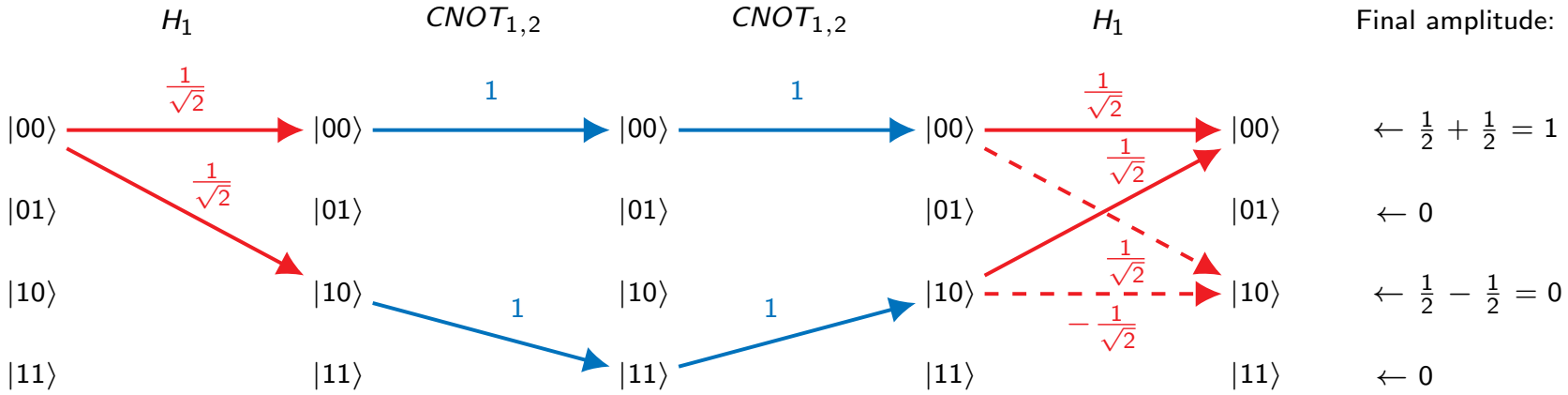

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Each satisfying assignment corresponds to a path! So WMC computes the path sum.

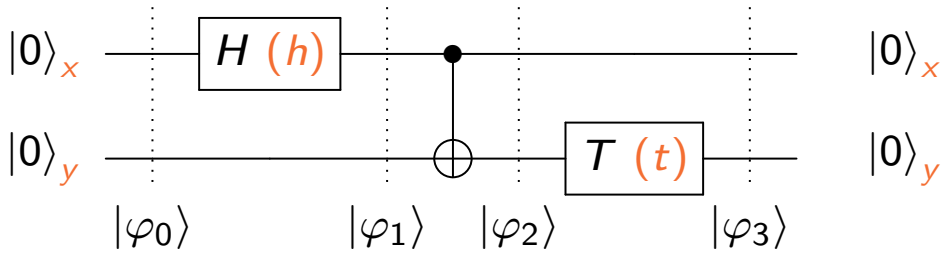
Roadmap

- SAT and #SAT (model counting)
- Encoding quantum circuits in logic
- Application: Simulation, Equivalence checking, Verification, Synthesis
- Different encodings and performance

Simulation

Simulation

Given an n-qubit quantum circuit C and a computational-basis measurement specification, which is described by a state $|\vec{b}\rangle$ on n qubits, compute the probability of measuring $|\vec{b}\rangle$ in state $C\,|0\ldots0\rangle$, i.e. $\langle\vec{b}|\,C\,|0\ldots0\rangle$.



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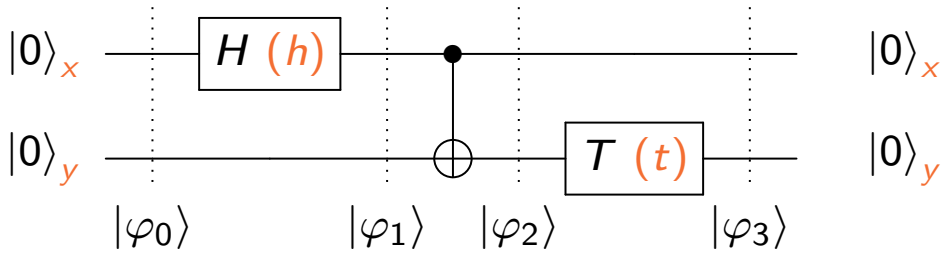

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We support general observables O by encoding $\langle 0|\,C^\dagger OC\,|0\rangle$.

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Verification

Quantum Hoare Logic [1]

Given a circuit C, projectors P and Q, the Hoare triple $\{P\}\ C\ \{Q\}$ is true if and only if for any input state $|\psi\rangle$, we have

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where $|\psi\rangle \models P$ iff $P\,|\psi\rangle = |\psi\rangle$.

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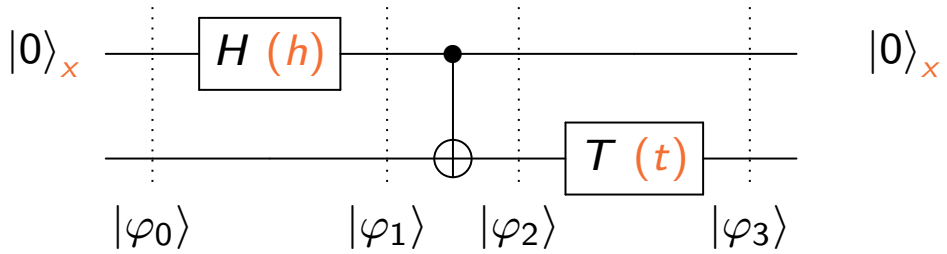

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[1] M. Ying, "Floyd–Hoare logic for quantum programs," *ACM Transactions on Programming Languages and Systems (TOPLAS)*, vol. 33, 2012.

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Definition (Approximate equivalence checking)

Let U and V be two n-qubit quantum circuits, Then U and V are ϵ-equivalent, written as $U \simeq_\epsilon V$, if and only if the **Jamiolkowski fidelity** between U and V is not less than $1 - \epsilon$ ($\epsilon \in [0, 1]$).

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We call this the "cyclic encoding" since it equals $F_{UV^\dagger}(\vec{q}_0, \ldots, \vec{q}_0)$

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Quantum circuit synthesis

The synthesis problem

Inputs:

- A circuit C_1 is gateset $\mathcal{G}_1$
- A gate set $\mathcal{G}_2$

Output:

- A minimum-depth circuit C_2 in gateset $\mathcal{G}_2$ that is equivalent to C_1

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The approximate synthesis problem

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Approach

quantum circuit synthesis $\longrightarrow$ maximum (weighted) model counting [1]

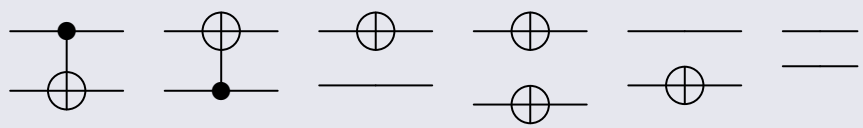
[1] D. Fremont, M. Rabe, and S. Seshia, "Maximum model counting," in *AAAI*, vol. 31, 2017.

Synthesis = Max#SAT over "which gate where"

Input $\mathcal{C}_1$ is given as $F_{\mathcal{C}_1}(\vec{x}, \vec{x}')$.

1. Encode one generic layer $F_{\mathcal{L}}(\vec{x}, \vec{x}', p)$ — selectors p *choose* the gate

e.g., for $p \in [P]$, the circuit $\mathcal{L}_p$ is:



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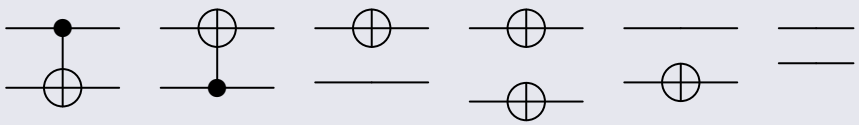

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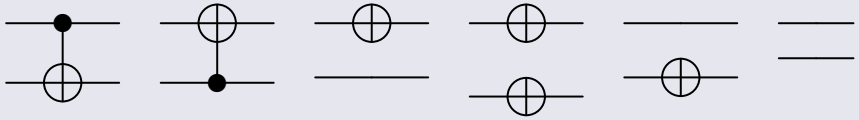

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1. Encode one generic layer $F_{\mathcal{L}}(\vec{x}, \vec{x}', p)$ — selectors p *choose* the gate

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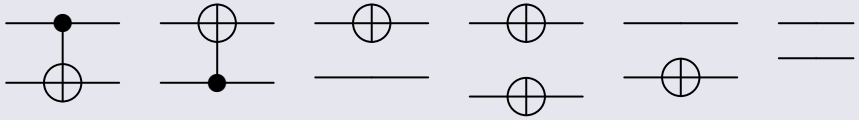

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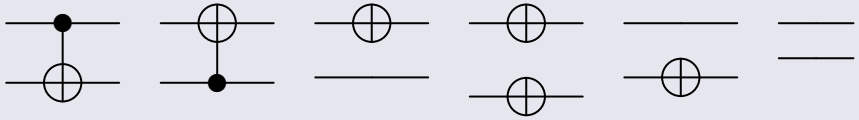

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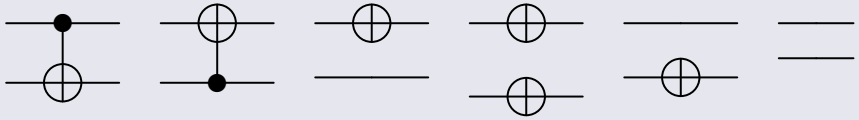

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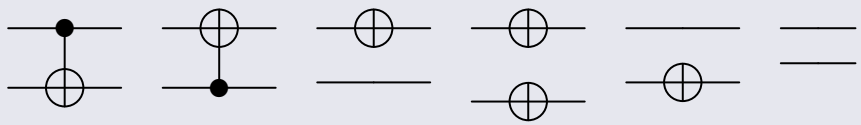

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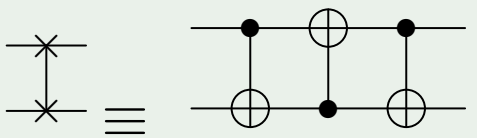
Exactly Synthesise `SWAP` from $\{CX\}$

d=3: count $= 2^2 = 4$,
$\mathcal{C}_2 = (CX_{01}, CX_{10}, CX_{01})$.



Roadmap

- SAT and #SAT (model counting)
- Encoding quantum circuits in logic
- Application: Simulation, Equivalence checking, Verification, Synthesis
- Different encodings and performance

All the other gates: negative & complex weights

Gate	Relation F_G	Weights
X, CX, *Toffoli*	Boolean	none
H	$h \Leftrightarrow (x \wedge x')$	$W(h) = -\frac{1}{\sqrt{2}}$
T	$(x \Leftrightarrow x') \wedge (w \Leftrightarrow x)$	$W(w) = \omega = \frac{1+i}{\sqrt{2}}$
S, $R_z(\theta)$	$(x \Leftrightarrow x') \wedge (w \Leftrightarrow x)$	$W(w) = i,\ e^{i\theta}$

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Different encoding in the Pauli basis (entanglement spectrum).

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Different encodings

In the Pauli basis: $\rho = \sum_j \beta_j \cdot P_j$ for $P_j \in \{ I, X, Y, Z \}^{\otimes n}$ for $\beta_j \in \mathbb{R}$,

We reserve two Boolean variables (x, z) for a Pauli, so e.g.:
$Z_1, Z_2 \equiv \overline{x}_1 \wedge \overline{x}_2 \wedge (z_1 \oplus z_2)$

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Pauli basis	$2n$	m	real (+negative)
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Encoding equivalence checking for n qubits:

Encoding Basis	Eq. Encoding	Performance
Pauli basis	Cyclic	+/-
Computational basis	Cyclic	++
Pauli basis	Choi iso. trick	++

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Practical performance

We can use any state-of-the-art weighted model counting which supports negative weights, such as Ganak [1]!

Random-circuit simulation comparison [2]:

- solid line: WMC
- dashed line: ZX graphical-calculus based simulator

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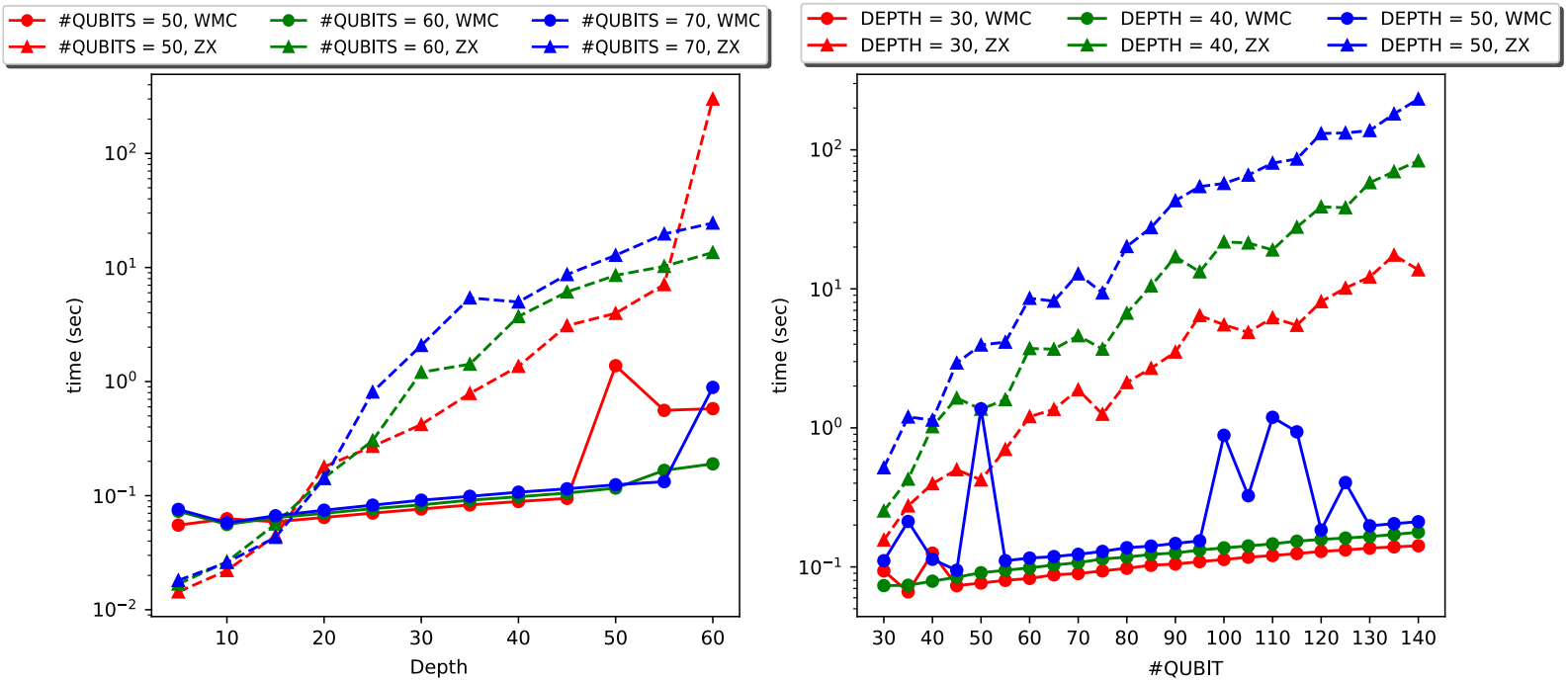

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Does synthesis work?

Depth-optimal TOFFOLI synthesis into $\{H, CX, C\sqrt{X}\}$

$$\text{Toffoli} = C\sqrt{X}_{0,2}\, C\sqrt{X}_{1,2}\, CX_{0,1}\, C\sqrt{X^\dagger}_{0,2}\, CX_{0,1}$$

iteration = depth	1	2	3	4	5	6
Clauses	136	992	1886	2780	3674	4568
Literals	524	3760	7126	10492	13858	17224
runtime (s)	0.03	0.04	0.16	3.39	141.86	577.78

converges to the optimal 6-layer circuit.

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Comparison with state-of-the-art `mitms`

$$CS = CX_{1,0}\, T_0^\dagger\, CX_{1,0}\, T_0\, T_1$$

depth d	3	4	5	6	7	8
`mitms` **[2]**	0.19	1.60	12.4	84.6	TO	TO
`Quokka#`	0.05	0.27	2.34	6.47	197	2354

reaches depths 7–8 where `mitms` times out.

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- New SAT-based methods for QC $\leftrightarrow$ new applications for solvers.
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Analyze your quantum circuits with Quokka#:

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Open problems

- Approximation algorithms for #SAT with negative weights.
- Support for sampling and noise.
- Push synthesis well beyond small qubit count and depth.
- Push open problems in Q. foundations: contextuality, MUB, non-locality, ...

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